Consider the following statements.

(i) $\{x\} \subseteq \{x, y, \{z\}\}$ (ii) $\{z\} \in \{x, y, \{z\}\}$ (iii) $\{z\} \subseteq \{x, y, \{z\}\}$

Which of the statements above are true ? **Circle the correct answer below.**

[a]   none are true   [b]   only (i) is true   [c]   only (ii) is true   [d]   only (iii) is true

[e]   only (i) and (ii) are true   [f]   only (i) and (iii) are true   [g]   only (ii) and (iii) are true   [h]   all are true

2

Rewrite the following statement using 2 variables and the formal structure mentioned in the 1.1 lecture notes.   SCORE: _____ / 2 PTS
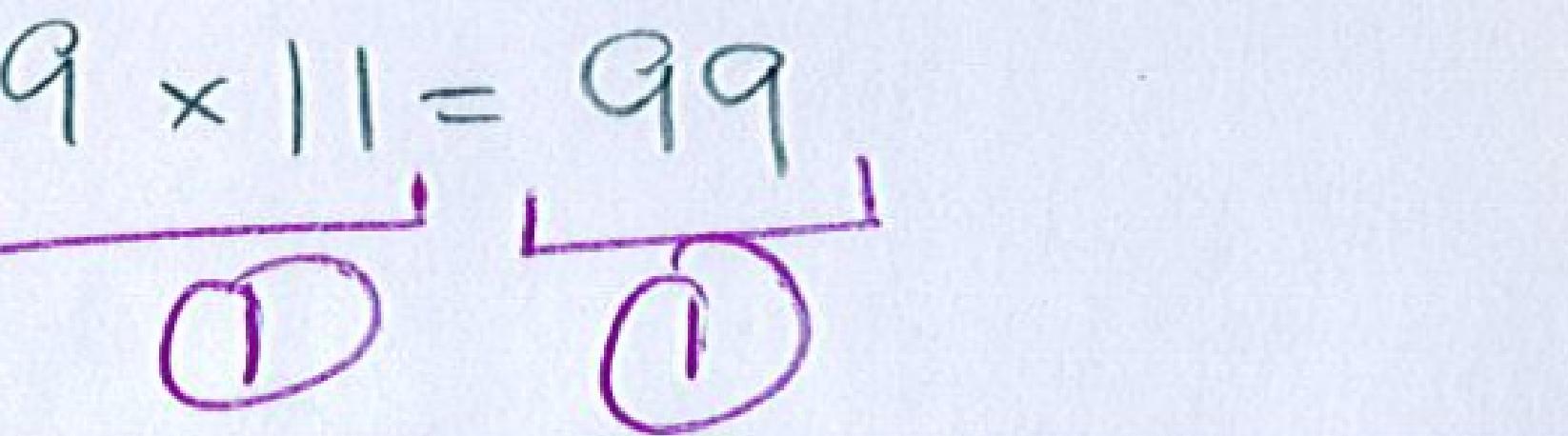
NOTE: Do NOT use any symbols, except for the variables.

"There's an instructor for each class."   ✓   THE ORDER IS IMPORTANT
FOR THIS PART ("FOR EACH" BEFORE
"THERE IS")

FOR EACH CLASS C, THERE IS A PERSON P ①

SUCH THAT P IS THE INSTRUCTOR FOR C ①

If $N = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and $L = \{a, b, c, d, e, f, g, h, i, j, k\}$,
how many elements are in the Cartesian product of L and N ?

$$9 \times 11 = 99$$

Fill in the blanks for the following **formal definitions**. Use proper mathematical notation.

[a] Given sets M and N, N is a subset of M (or $N \subseteq M$) if and only if $\text{FOR ALL } x \in N, \; x \in M$

[b] The Cartesian product of sets M and N is $M \times N = \{(x, y) \mid x \in M \text{ AND } y \in N\}$

Determine if $p \oplus q \equiv \sim p \leftrightarrow q$. **State your final answer clearly.**

P	q	$p \oplus q$ ①	$\sim p$ ($\frac{1}{2}$)	$\sim p \leftrightarrow q$ ①
T	T	F	F	F
T	F	T	F	T
F	T	T	T	T
F	F	F	T	F

($\frac{1}{2}$)

YES, $p \oplus q \equiv \sim p \leftrightarrow q$

($-\frac{1}{2}$) FOR EACH ERROR IN TRUTH TABLE (MINIMUM

SORE = 0)

Write the **<u>formal definition</u>** of a relation used in discrete math. Use correct English and mathematical notation.   SCORE: _____ / 2 PTS

A RELATION FROM SET A TO SET B IS A SUBSET OF $A \times B$

GRADED BY ME

Classify each statement as Universal Existential (**UE**), Existential Universal (**EU**) or Universal Conditional (**UC**).   **SCORE:** ___ / 2 POINTS

[a]   All calculus students have passed the same placement test.   EU ①

[b]   Students who have parking stickers can park in lot C.   UC ①

Consider the statement "if $\frac{1}{x} < 1$, then $x > 1$". (Assume x is a particular real number.)

$p \longrightarrow q$

$\underbrace{\phantom{\frac{1}{x}<1}}_{p} \quad \underbrace{}_{q}$

SCORE: ___ / 4 POINTS

[a] Write a logically equivalent statement using "is necessary for". **Do NOT use statement variables in your final answer.**

① $x > 1$ IS NECESSARY FOR $\frac{1}{x} < 1$

m IS NECESSARY FOR n
n → m

[b] Write a logically equivalent statement using "unless". **Do NOT use statement variables in your final answer.**

① $x > 1$ UNLESS $\frac{1}{x} \not< 1$

m UNLESS n
~n → m

[c] Write the contrapositive of the statement. **Do NOT use statement variables in your final answer.**

① IF $x \not> 1$ THEN $\frac{1}{x} \not< 1$

OK IF YOU SAID "$\frac{1}{x} \geq 1$" INSTEAD OF "$\frac{1}{x} \not< 1$"
AND "$x \leq 1$"

[d] Write the negation of the statement. **Do NOT use statement variables in your final answer.**

① $\frac{1}{x} < 1$ AND $x \not> 1$

INSTEAD OF "$x \not> 1$"

$\sim(p \to q) \equiv \sim(\sim p \vee q)$
$\equiv p \wedge \sim q$

Determine if the following argument is valid. <u>State your final answer clearly.</u>   SCORE: _____ / 8 PTS

NOTES: This is NOT an essay question. Use the method shown in lecture and section 2.3. Do NOT use the Rules of Inference.

If I save a lot of money, then I can quit my second job or I can buy a new car.

p q r

I did not save a lot of money and I cannot quit my second job.

Therefore, I cannot buy a new car.

$(-\frac{1}{2})$ FOR EACH ERROR IN TRUTH TABLE (MINIMUM=0) SCORE

$p \rightarrow (q \vee r)$

$\sim p \wedge \sim q$

$\therefore \sim r$

②

① FOR IDENTIFYING WHICH WERE CRITICAL ROWS (LAST 2)

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim r$
T	T	T	T	T	F	F	F	F
T	T	F	T	T	F	F	F	T
T	F	T	T	T	F	T	F	F
T	F	F	F	F	F	T	F	T
F	T	T	T	T	T	F	F	F
F	T	F	T	T	T	F	F	T
F	F	T	T	(T)	T	T	(T)	(F)
F	F	F	F	(T)	T	T	(T)	(T)

① INVALID

Ⓔ } CRITICAL ROWS
T